Cosmology via Metric-Independent Volume-Form Dynamics

Eduardo Guendelman and Emil Nissimov and Svetlana Pacheva

Abstract The method of non-Riemannian volume-forms (metric-independent co-variant integration measure densities on the spacetime manifold) is applied to construct a unified model of dynamical dark energy plus dark matter as a dust fluid resulting from a hidden Noether symmetry of the pertinent scalar field Lagrangian. Canonical Hamiltonian treatment and Wheeler-DeWitt quantization of the latter model are briefly discussed.

1 Introduction

Alternative spacetime volume-forms (generally-covariant integration measure densities) independent on the Riemannian metric on the pertinent spacetime manifold have profound impact in any field theory models with general coordinate reparametrization invariance, such as general relativity and its extensions, strings and (higher-dimensional) membranes [12, 10, 11, 14].

The principal idea is to replace or employ alongside the standard Riemannian integration density given by $\sqrt{-g}$ (square root of the determinant $g = \det\|g_{\mu\nu}\|$ of the Riemannian metric $g_{\mu\nu}$) one or more non-Riemannian (metric-independent) covariant integration measure densities defined in terms of dual field-strengths $\Phi(B)$ of auxiliary maximal rank antisymmetric tensor gauge fields $B_{\mu\nu\lambda}$:

Eduardo Guendelman
Department of Physics, Ben-Gurion University of the Negev, Beer-Sheva, Israel e-mail: guendel@bgu.ac.il

Emil Nissimov
Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Sofia, Bulgaria e-mail: nissimov@inrne.bas.bg

Svetlana Pacheva
Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Sofia, Bulgaria e-mail: svetlana@inrne.bas.bg

$$\Phi(B) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu B_{\nu\kappa\lambda} \; , \tag{1}$$

The corresponding non-Riemannian-modified-measure gravity-matter models were called "two-measure (gravity) theories" and the associated auxiliary tensor gauge fields $B_{\mu\nu\lambda}$ – "measure gauge fields".

The auxiliary "measure" gauge fields trigger a number of physically interesting phenomena:

- The equations of motion w.r.t. $B_{\mu\nu\lambda}$ produce dynamical constraints involving *arbitrary integration constants*, where one of the latter *always* acquires the meaning of a *dynamically generated cosmological constant*.
- Employing the canonical Hamiltonian formalism for Dirac-constrained systems we find that $B_{\mu\nu\lambda}$ are in fact almost pure gauge degrees of freedom except for the above mentioned arbitrary integration constants which are identified with the conserved Dirac-constrained canonical momenta conjugated to the "magnetic" components (B_{ijk}) of the "measure" gauge fields.
- Upon applying the non-Riemannian volume-form formalism to minimal $N = 1$ supergravity the dynamically generated cosmological constant triggers spontaneous supersymmetry breaking and mass generation for the gravitino (supersymmetric Brout-Englert-Higgs effect) [16]. Applying the same formalism to anti-de Sitter supergravity allows to produce simultaneously a very large physical gravitino mass and a very small *positive* observable cosmological constant [16] in accordance with modern cosmological scenarios for slowly expanding universe of the present epoch [21, 20, 22].
- Employing two independent non-Riemannian volume-forms like (1) in generalized gravity-gauge+scalar-field models [17], thanks to the appearance of several arbitrary integration constants through the equations of motion w.r.t. the "measure" gauge fields, we obtain in the physical "Einstein-frame" a remarkable effective scalar potential with two infinitely large flat regions (for large negative and large positive values of the scalar field φ) with vastly different scales appropriate for a unified description of both the early and late universe' evolution. Another remarkable feature is the existence of a stable initial phase of *non-singular* universe creation preceding the inflationary phase – stable "emergent universe" without "Big-Bang" [17].

As a specific illustration of the usefulness of the non-Riemannian volume-form method and extending the study in [15, 1] we discuss a modified gravity+single-scalar-field model where the scalar Lagrangian couples symmetrically both to the standard Riemannian volume-form given by $\sqrt{-g}$ as well as to another non-Riemannian volume-form (1). The pertinent scalar field dynamics provides a unified description of both dark energy via dynamical generation of a cosmological constant, and dark matter as a "dust" fluid with geodesic flow as a result of a hidden Noether symmetry. Further, we briefly consider the canonical Hamiltonian treatment and the Wheeler-DeWitt quantization of the above unified dark energy plus dust fluid dark matter model.

2 Dark Energy and Dust Fluid Dark Matter via Non-Riemannian Volume-Form Dynamics

We will consider the following non-conventional gravity+scalar-field action – a particular case of the general class of the "two-measure" gravity-matter theories [12, 10, 11] (for simplicity we use units with the Newton constant $G_N = 1/16\pi$):

$$S = \int d^4x \sqrt{-g} R + \int d^4x \left(\sqrt{-g} + \Phi(B) \right) L(\varphi, X) . \tag{2}$$

Here $\Phi(B)$ is as in (1) and $L(\varphi, X)$ is general-coordinate invariant Lagrangian of a single scalar field $\varphi(x)$ of a generic "k-essence" form [8, 2] (i.e., a nonlinear (in general) function of the scalar kinetic term X): $L(\varphi, X) = \sum_{n=1}^{N} A_n(\varphi) X^n - V(\varphi)$, $X \equiv -\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi$. The energy-momentum tensor corresponding to (2) reads:

$$T_{\mu\nu} = g_{\mu\nu} L(\varphi, X) + \left(1 + \frac{\Phi(B)}{\sqrt{-g}} \right) \frac{\partial L}{\partial X} \partial_\mu \varphi \partial_\nu \varphi . \tag{3}$$

The essential new feature is the dynamical constraint on the scalar Lagrangian, which results from the equation of motion w.r.t. "measure" gauge field $B_{\mu\nu\lambda}$:

$$\partial_\mu L(\varphi, X) = 0 \quad \longrightarrow \quad L(\varphi, X) = -2M = \text{const} , \tag{4}$$

where M is an *arbitrary integration constant*. We will take $M > 0$ in view of its interpretation as a *dynamically generated cosmological constant* (see (7) below).

A remarkable property of the scalar field action in (2) is the presence of a hidden Noether symmetry of the latter under the nonlinear transformations:

$$\delta_\varepsilon \varphi = \varepsilon \sqrt{X} , \; \delta_\varepsilon g_{\mu\nu} = 0 , \; \delta_\varepsilon B_{\mu\nu\lambda} = -\varepsilon \frac{1}{2\sqrt{X}} \varepsilon_{\mu\nu\lambda\kappa} g^{\kappa\rho} \partial_\rho \varphi \left(\Phi(B) + \sqrt{-g} \right) . \tag{5}$$

The standard Noether procedure yields the conserved current:

$$\nabla_\mu J^\mu = 0 \quad , \quad J^\mu \equiv \left(1 + \frac{\Phi(B)}{\sqrt{-g}} \right) \sqrt{2X} g^{\mu\nu} \partial_\nu \varphi \frac{\partial L}{\partial X} . \tag{6}$$

Let us stress that the existence of the hidden symmetry (5) of the action (2) *does not* depend on the specific form of the scalar field Lagrangian.

Now, $T_{\mu\nu}$ (3) and J^μ (6) can be rewritten in a relativistic hydrodynamical form (taking into account (4)):

$$T_{\mu\nu} = \rho_0 u_\mu u_\nu - 2M g_{\mu\nu} \quad , \quad J^\mu = \rho_0 u^\mu , \tag{7}$$

where:

$$\rho_0 \equiv \left(1 + \frac{\Phi(B)}{\sqrt{-g}} \right) 2X \frac{\partial L}{\partial X} \quad , \quad u_\mu \equiv \frac{\partial_\mu \varphi}{\sqrt{2X}} \quad (u^\mu u_\mu = -1) . \tag{8}$$

For the pressure p and energy density ρ we obtain:

$$p = -2M = \text{const} \quad , \quad \rho = \rho_0 - p = 2M + \left(1 + \frac{\Phi(B)}{\sqrt{-g}}\right) 2X \frac{\partial L}{\partial X} , \qquad (9)$$

wherefrom indeed the integration constant M appears as *dynamically generated cosmological constant*. Moreover the covariant energy-momentum conservation $\nabla^\nu T_{\mu\nu} = 0$, due to the constancy of the pressure (first Eq.(9)), actually implies *both* the conservation of the Noether current J^μ (6) as well as the *geodesic flow* equation: $u_\nu \nabla^\nu u_\mu = 0$.

The above results lead to the following interpretation in accordance with the standard Λ-CDM model (see e.g. [9]). The energy-momentum tensor (7) consists of two parts:

- Dark energy part given by the second cosmological constant term in $T_{\mu\nu}$ (7), which arises due to the dynamical constraint on the scalar field Lagrangian (4) with $p_{\text{DE}} = -2M$, $\rho_{\text{DE}} = 2M$ (cf. Eqs.(9)).
- Dark matter part given by the first term in (7) (cf. also (9)) with $p_{\text{DM}} = 0$, $\rho_{\text{DM}} = \rho_0$ (ρ_0 as in (8)). The latter describe a dust fluid with dust "particle number" conservation (6) and flowing along geodesics.

The idea of unified description of dark energy and dark matter is the subject of numerous earlier papers exploiting a variety of different approaches. Among them are generalized Chaplygin gas models [5, 19], "mimetic" dark matter models [6, 7], constant pressure ansatz models [3] *etc.*

3 Canonical Hamiltonian Formalism and Wheller-DeWitt Equation

For a systematic canonical Hamiltonian treatment of gravity-matter models based on metric-independent volume-forms we refer to [13] and specifically to the second reference therein for the full Hamiltonian treatment of the present model (2). Here, for simplicity, we will consider a reduction of (2) where the spacetime metric is taken of the Friedmann-Lemaitre-Robinson-Walker (FLRW) class:

$$ds^2 = -N^2(t)dt^2 + a^2(t)\left[\frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] , \qquad (10)$$

and where φ and the "measure" gauge field B are taken to depend only on t. The reduced action resulting from (2) reads (taking the standard form of the scalar Lagrangian):

$$S = 6 \int dt N a^3 \left[-\frac{1}{N^2}\left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2}\right] + \int dt \left(\partial_t B + N a^3\right)\left(\frac{1}{2N^2} \dot{\varphi}^2 - V(\varphi)\right) . \quad (11)$$

The equation of motion w.r.t. B produces the dynamical constraint (reduced form of (4)) with explicit solution for $\varphi(t)$:

$$\dot{\varphi}^2 = 2\big(V(\varphi) - 2M\big) \;\longrightarrow\; \int_{\varphi(0)}^{\varphi(t)} \frac{d\varphi}{\sqrt{2\big(V(\varphi) - 2M\big)}} = \pm t. \tag{12}$$

The hidden "dust" Noether symmetry (cf. (5) and (6)) of the reduced action (11) now takes the form:

$$\delta_\varepsilon \varphi = \varepsilon \frac{\dot{\varphi}}{N} \;,\;\; \delta_\varepsilon B = \varepsilon \frac{1}{N}\big(\partial_t B + Na^3\big) \;,\;\; \delta_\varepsilon a = 0 \;\;,\;\; \frac{d}{dt}\left[\big(Na^3 + \partial_t B\big)\frac{\dot{\varphi}^2}{N^3}\right] = 0 \,. \tag{13}$$

The canonical Hamiltonian treatment a'la Dirac of the reduced action (11) yields the following Dirac-constrained Hamiltonian (N appearing as a Lagrange multiplier of the first class constraint in the brackets):

$$\mathscr{H}_{\text{total}} = N\left[-\frac{p_a^2}{24a} - 6Ka - \pi_B a^3 + \sqrt{2\big(V(\varphi) + \pi_B\big)}\, p_\varphi\right], \tag{14}$$

where p_a and π_B are the canonically conjugated momenta of a and B, respectively.

The quantum Wheeler-DeWitt equation corresponding to (14) is significantly simplified upon changing variables as:

$$a \to \widetilde{a} = \frac{4}{\sqrt{3}} a^{3/2} \;\;,\;\; \varphi \to \widetilde{\varphi} = \int \frac{d\varphi}{\sqrt{2\big(V(\varphi) - 2M\big)}} \,, \tag{15}$$

where from (12) we find that the new scalar field coordinate $\widetilde{\varphi}$ will have the meaning of a (cosmic) time. Since B turns out to be a cyclic variable in (14) the quantized canonical momentum $\widehat{\pi}_B = -i\delta/\delta B$ is immediately diagonalized whose eigenvalues are denoted by $\pi_{\mathscr{B}} = -2M$, so that M will have the meaning of a dynamically generated cosmological constant. Further, we notice that the quantized form of the last term in (14), which is the Hamiltonian expression for the conserved "dust" Noether symmetry charge (13), will simplify to $\sqrt{2\big(V(\varphi) + \pi_B\big)}\left(-i\frac{d}{d\varphi}\right) = -id/d\widetilde{\varphi}$ and is straightforwardly diagonalized with eigenvalues $\mathscr{E}$. Accordingly, the total Wheeler-DeWitt wave function will have the form $\psi(a, \varphi, B) = \psi_{\text{grav}}(\widetilde{a})\, e^{i\mathscr{E}\widetilde{\varphi} - i2MB}$ (with $\widetilde{a}$ and $\widetilde{\varphi}$ as in (15)), and the Wheeler-DeWitt equation reduces to "energy" eigenvalue Schrödinger equation for the gravitational part of the total wave function:

$$\left[-\frac{1}{2}\frac{\partial^2}{\partial\widetilde{a}^2} - \frac{3}{8}M\widetilde{a}^2 + 6K\big(\frac{\sqrt{3}}{4}\widetilde{a}\big)^{2/3} - \mathscr{E}\right]\psi_{\text{grav}}(\widetilde{a}) = 0 \tag{16}$$

In the special case of zero spacial curvature $K = 0$ in the FLRW metric (10) Eq.(16) reduces to the energy eigenvalue Schrödinger equation for the *inverted* harmonic oscillator [4] with negative frequency squared $\omega^2 \equiv -\frac{3}{4}M$ (the dynamically generated cosmological constant M must be positive).

In particular, the inverted oscillator was applied in [18] to study the quantum mechanical dynamics of the scalar field in the so called "new inflationary" scenario. Since the energy eigenvalue spectrum of the inverted harmonic oscillator is continuous ($\mathscr{E} \in (-\infty, +\infty)$) and the corresponding energy eigenfunctions are not square-integrable, its application in the context of cosmology [18] requires employment of wave-packets instead of energy eigenfunctions.

References

1. S. Ansoldi and E. Guendelman, *JCAP* **1305**, 036 (2013) (*arXiv:1209.4758*).
2. C. Armendariz-Picon, V. Mukhanov and P. Steinhardt, *Phys. Rev. Lett.* **85**, 4438 (2000) (*arXiv:astro-ph/0004134*); *Phys. Rev.* **D63**, 103510 (2001) (*arXiv:astro-ph/0006373*).
3. A. Aviles, N. Cruz, J. Klapp and O. Luongo, *Gen. Rel. Grav.* **47**, art.63 (2015) (*arXiv:1412.4185*).
4. G. Barton, *Ann. Phys.* **166**, 322-363 (1986).
5. N. Bilic, G. Tupper, and R. Viollier, *Phys. Lett.* **535B**, 17 (2002) (*arXiv:astro-ph/0111325*); *Phys. Rev.* **D80**, 023515 (2009) (*arXiv:0809.0375*); *J. Phys.* **A40**, 6877 (2007) (*arXiv:gr-qc/0610104*).
6. A. Chamseddine and V. Mukhanov, *JHEP* **1311**, 135 (2013) (*arXiv:1308.5410*).
7. A. Chamseddine, V. Mukhanov and A. Vikman, *JCAP* **1406**, 017 (2014) (*arXiv:1403.3961*).
8. T. Chiba, T.Okabe and M. Yamaguchi, *Phys. Rev.* **D62**, 023511 (2000) (*arXiv:astro-ph/9912463*).
9. J. Frieman, M. Turner and D. Huterer, *Ann. Rev. Astron. Astrophys.* **46**, 385-432 (2008) (*arXiv:0803.0982*).
10. E.I. Guendelman, *Mod. Phys. Lett.* **A14**, 1043-1052 (1999) (*arXiv:gr-qc/9901017*).
11. E.I. Guendelman, *Found. Phys.* **31**. 1019-1037 (2001) (*arXiv:hep-th/0011049*).
12. E.I. Guendelman and A. Kaganovich, *Phys. Rev.* **D55**, 5970 (1997) (*arXiv:gr-qc/9611046*); *Phys. Rev.* **D60**, 065004 (1999) (*arXiv:gr-qc/9905029*).
13. E.I. Guendelman, E. Nissimov and S. Pacheva, *Int. J. Mod. Phys.* **A30**, 1550133 (2015) (*arXiv:1504.01031*); *arXiv:1508.02008.*
14. E. Guendelman, A. Kaganovich, E. Nissimov and S. Pacheva, *Phys. Rev.* **D66**, 046003 (2002) (*arXiv:hep-th/0203024*).
15. E. Guendelman, D. Singleton and N. Yongram, *JCAP* **1211**, 044 (2012) (*arXiv:1205.1056*).
16. E. Guendelman, E. Nissimov, S. Pacheva and M. Vasihoun, *Bulg. J. Phys.* **41**, 123-129 (2014) (*arXiv:1404.4733*); in *Eight Mathematical Physics Meeting*, ed. by B. Dragovic and I. Salom (Belgrade Inst. Phys. Press, Belgrade, 2015), pp.105-115 (*arXiv:1501.05518*).
17. E. Guendelman, R. Herrera, P. Labrana, E. Nissimov and S. Pacheva, *Gen. Rel. Grav.* **47**, art.10 (2015) (*arXiv:1408.5344v4*).
18. A. Guth and S.-Y. Pi, *Phys. Rev.* **D32**, 1899-1920 (1985).
19. A.Yu. Kamenshchik , U. Moschella and V. Pasquier, *Phys. Lett.* **511B**, 265 (2001) (*arXiv:gr-qc/0103004*).
20. S. Perlmutter, *et.al.*, *Astrophys. J.* **517**, 565-586 (1999) (*arXiv:astro-ph/9812133*).
21. A.G. Riess, *et.al.*, *Astron. J.* **116**, 1009-1038 (1998) (*arXiv:astro-ph/9805201*).
22. A.G. Riess, *et.al.*, *Astrophys. J.* **607**, 665-687 (2004) (*arXiv:astro-ph/0402512*).